

CURVES

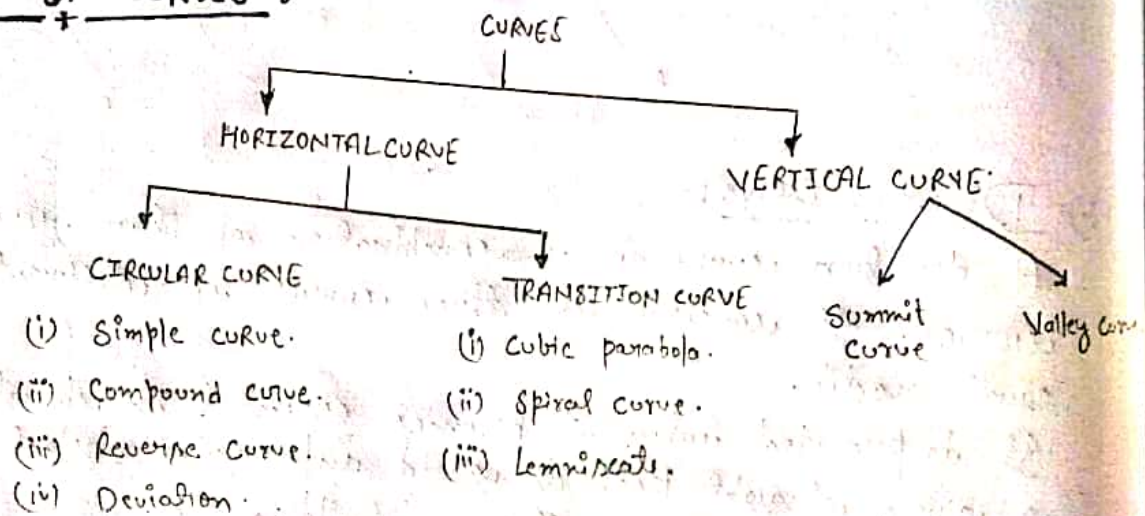
COMPONENTS OF HIGHWAY DESIGN:-

Horizontal Alignment →

Vertical Alignment →

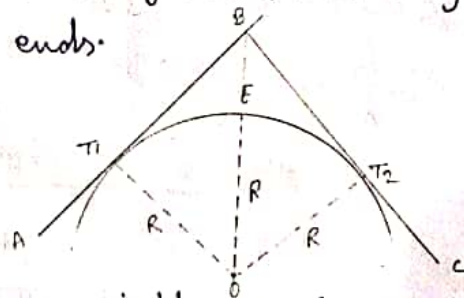


TYPES OF CURVES:-



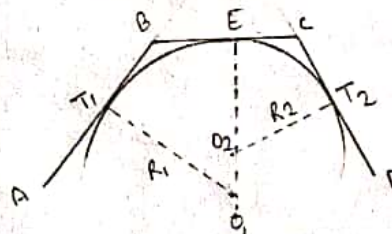
(i) Simple curve-

It is a curve which consists of a single arc of constant radius connecting the two straight. It is tangential to the two straight at the joining ends.



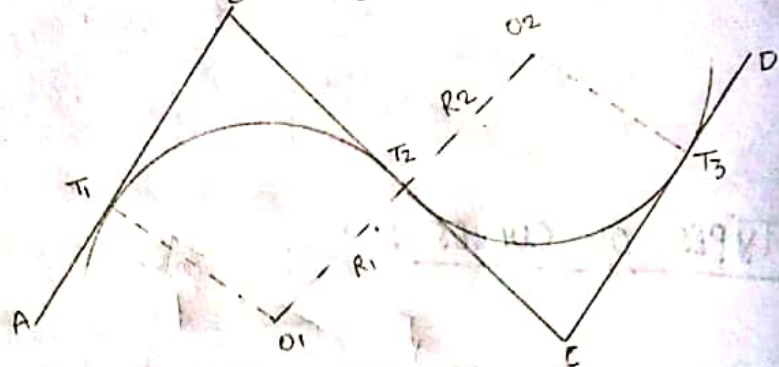
(ii) Compound curve-

It consists of two or more simple circular curves of different radii turning or bending in the same direction and lying on the same side of common tangent. Their centres lie on the same side of the curve.



Reverse / serpentine curve:-

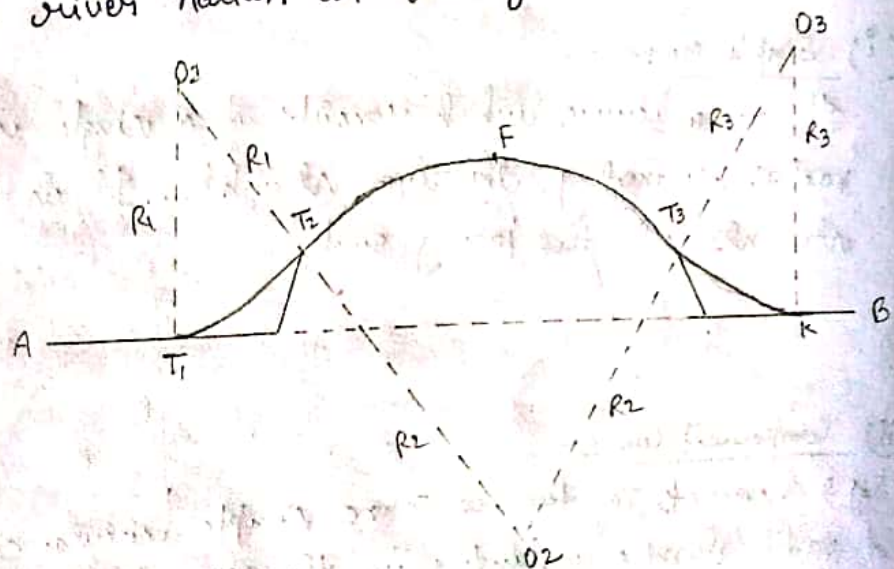
It is a curve made of two arcs having equal or different radii bending in opposite directions with a common tangent at their junction. Their centres lie on the opposite sides of the curve. Reverse curves are used when the straight lines are parallel or intersect at a very small angle. They are commonly used in railways sidings and sometimes on railway tracks and roads meant for low speeds.



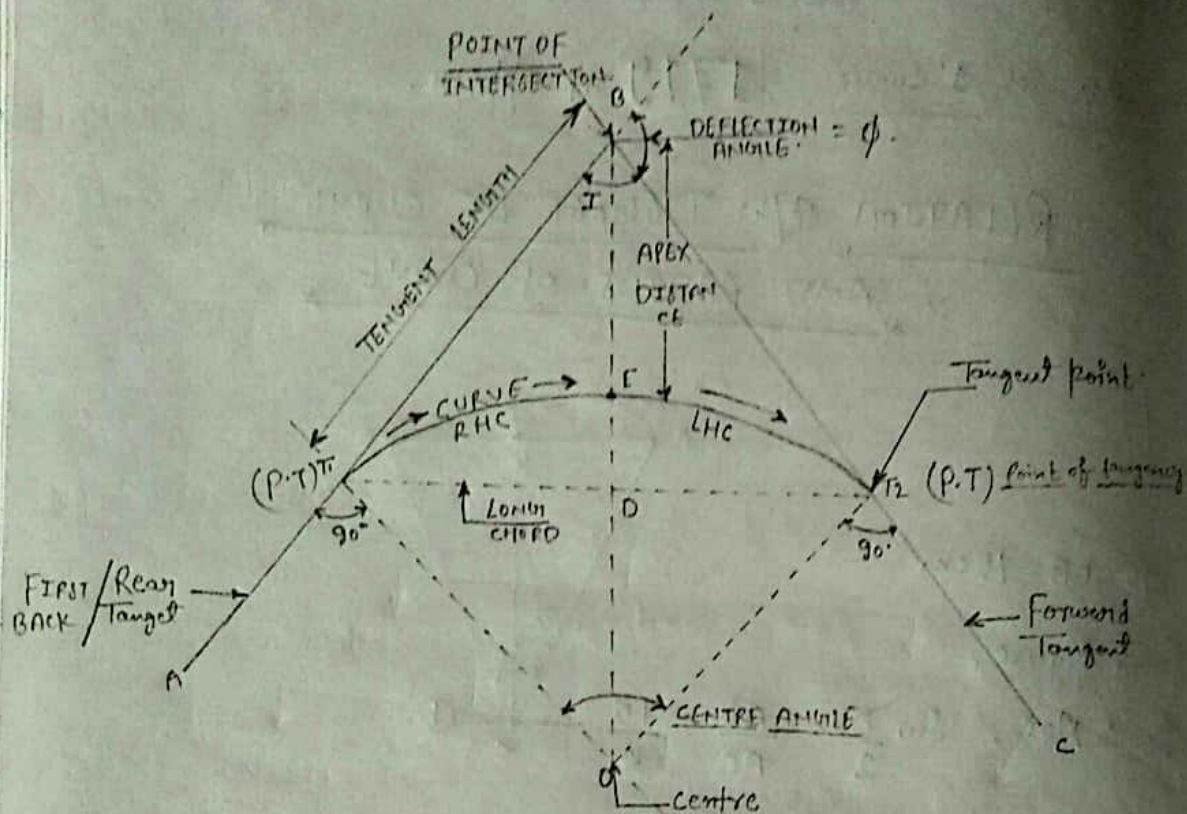
(2) Deviation Curve:-

A deviation curve is a combination of two reverse curves. It consists of three curves of different or equal radii.

It is provided when it becomes essential to deviate from a given straight path to avoid intervening obstacle such as bend of river, nallah or building etc.



Various parts of a Curve :-



DE = Mid ordinate.

BE = Apex distance.

E = Apex point.

Normal chord $\rightarrow$ A chord b/w two successive regular station on a curve is known as normal chord.

Sub chord $\rightarrow$ Any chord smaller than normal chord is called sub chord.

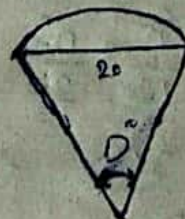
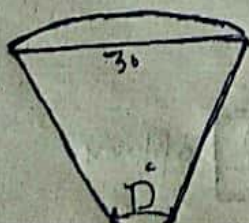
A curve may be designated either by radius or by the angle subtended at the centre by a chord of particular length.

DESIGNATION OF CURVE

★ Radius is adopted in U.K and Australia.

★ Later India, Canada, U.S.A.

$\rightarrow$ In India a curve is designated by an angle in degrees subtended at the centre by a chord of 30 m or 20 m. [100ft]



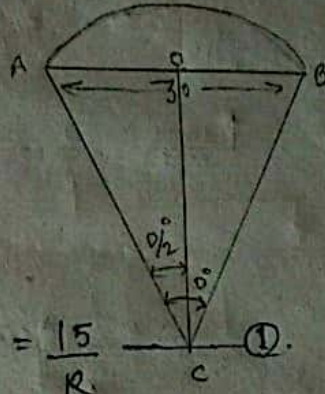
Degree of curve

The angle subtended at the centre of curve by a unit chord of 30m.

Radius of 1° curve

1719 m approx.

RELATION B/W DEGREE OF CURVE
AND RADIUS OF CURVE



$$AO = OB = 15 \text{ cm}$$

from figure,

$$\text{In } \triangle AOC, \sin \frac{D}{2} = \frac{AO}{AC} = \frac{15}{R} \quad \text{①}$$

$$\Rightarrow R = \frac{15}{\sin \frac{D}{2}}$$

When D is small $\sin \frac{D}{2}$ may be assumed approx equal to $\frac{D}{2}$ in radian.

$$\text{i.e., } \sin \frac{D}{2} = \frac{D}{2} \text{ radian.}$$

Convert $\frac{D}{2}$ in radian.

$$\Rightarrow 180^\circ = \pi \text{ radian.}$$

$$\Rightarrow 1^\circ = \frac{\pi}{180} \text{ radian.}$$

$$\Rightarrow \frac{D}{2}^\circ = \frac{\pi}{180} \times \frac{D}{2}$$

By putting the values in eqⁿ ①

$$\Rightarrow R = \frac{15}{\frac{\pi}{180} \times \frac{D}{2}}$$

$$\Rightarrow \frac{15 \times 360}{\pi D} \Rightarrow \boxed{R = \frac{1719}{D}} \text{ approx.}$$

for 20 chord,

$$\Rightarrow \sin \frac{D}{2} = \frac{10}{R}$$

$$\Rightarrow R = \frac{10}{\sin \frac{D}{2}}$$

$$\Rightarrow R = \frac{10 \times 360^\circ}{\pi D}$$

$$\Rightarrow \boxed{R = \frac{1146}{D}} \text{ approx.}$$

~~Ques.~~ ELEMENTS OF SIMPLE CIRCULAR CURVE

1. Angle of deflection + Angle of I = 180°
 $\angle \phi + \angle I = 180^\circ$

$$\therefore \boxed{\angle \phi = 180^\circ - \angle I}$$

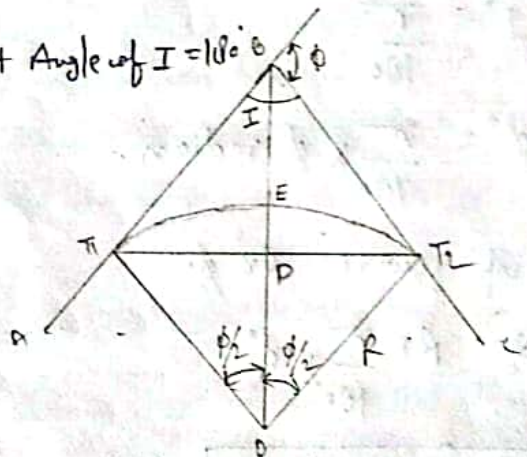
2. Tangent $T_1B = T_2B$.

In Right angle ΔOT_1D .

$$\frac{OT_1}{OD} = \tan \frac{\phi}{2}$$

$$OT_1 = OD \tan \frac{\phi}{2}$$

$$\boxed{\text{Tangent length} = R \tan \frac{\phi}{2}}$$



Length of long chord $T_1 D T_2 = L$.

In Right Angle $\Delta T_1 D O$

$$\sin \frac{\phi}{2} = \frac{T_1 D}{T_1 O}$$

$$\Rightarrow T_1 D = T_1 O \sin \frac{\phi}{2}$$

$$\Rightarrow T_1 D = R \sin \frac{\phi}{2}$$

$$\boxed{\text{Length of long chord} = 2 \times R \sin \frac{\phi}{2}}$$

Length of curve $= l = T_1 E T_2$.

$$l = R \times \phi \text{ in radian.}$$

$$180^\circ = \pi \text{ radian}$$

$$1^\circ = \frac{\pi}{180}$$

$$\phi^\circ = \frac{\pi}{180} \times \phi \text{ radian.}$$

Length of curve $= R \times \phi$.

$$\Rightarrow R \times \frac{\pi}{180} \times \phi^\circ$$

$$\boxed{\text{Length of curve} = \frac{R \pi \phi^\circ}{180}}$$

$\therefore \Delta T_1 O D$

$$\cos \frac{\phi}{2} = \frac{D O}{T_1 O}$$

$$\Rightarrow D O = R \cos \frac{\phi}{2}$$

$$\Rightarrow E O = E O - D O$$

$$\Rightarrow E O = R - R \cos \frac{\phi}{2}$$

$$\Rightarrow E O = R \left[1 - \cos \frac{\phi}{2} \right]$$

$$\boxed{\text{Versed Sine} = R \left[1 - \cos \frac{\phi}{2} \right]}$$

6. In ΔBOT

$$\Rightarrow \cos \frac{\phi}{2} = \frac{OT}{BO}$$

$$\Rightarrow BO = \frac{OT}{\cos \frac{\phi}{2}} \Rightarrow \frac{R \cos \phi}{2}$$

$$\Rightarrow BE = BO - EO$$

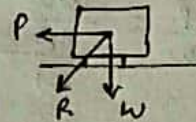
$$\Rightarrow BE = \frac{R \cos \phi}{2} - R$$

$$\boxed{\text{Apex distance} = R \left[\cos \frac{\phi}{2} - 1 \right]}$$

SUPER ELEVATION.

When a vehicle is moving on a straight levelled portion of road or railway the force acting on the vehicle is its own weight. But when a vehicle passes from straight path to curved path, the different forces act on it:-

- weight of vehicle.
- Centrifugal force.



The centrifugal force always acts in a direction $\perp$ to the axis of rotation. Since the axis of rotation is vehicle, centrifugal force acts horizontal and therefore the vehicle tends to push away from the centre of curve.

To counteract this effect, plane of road or railway surface is made perpendicular to the resultant of centrifugal forces and weight of vehicle by raising the outer edge of road or outer rail of rail track.

Thus, this raising of outer edge of road or track over the inner one on curves is called superelevation.

Purpose:-

- To prevent excessive wear on the outer wheel than inner one.
- To equalize press. on both wheel.
- To prevent the vehicle from overturning.

DERIVATION FOR SUPER ELEVATION

Centrifugal force

$$= \frac{Wv^2}{gR}$$

g = acceleration due to gravity.

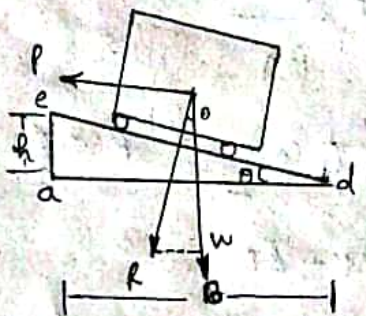
b = width of road.

R = gauge or rail in (m)

v = speed.

h = height of super elevation.

θ = Inclination of road.



At equilibrium, $R, W, \& P$ must be equal.

$$\tan \theta = \frac{ca}{ad} = \frac{P}{W}$$

$$\tan \theta = \frac{h}{b} = \frac{P}{W}$$

$$\tan \theta = \frac{h}{b}$$

$$h = b \tan \theta$$

$$\tan \theta = \frac{P}{W} = \frac{\frac{Wv^2}{gR}}{W}$$

$$h = b \times \frac{v^2}{gR}$$

$$\therefore h = \frac{bv^2}{gR} \text{ for road}$$

$$h = \frac{Gv^2}{gR} \text{ for railway.}$$

CENTRIFUGAL RATIO :-

Ratio of centrifugal force and the weight.

$$\frac{P}{W}$$

* It is taken as $\frac{1}{4}$ for road and $\frac{1}{8}$ for railway.

• for road : $v = \sqrt{\frac{gR}{4}}$

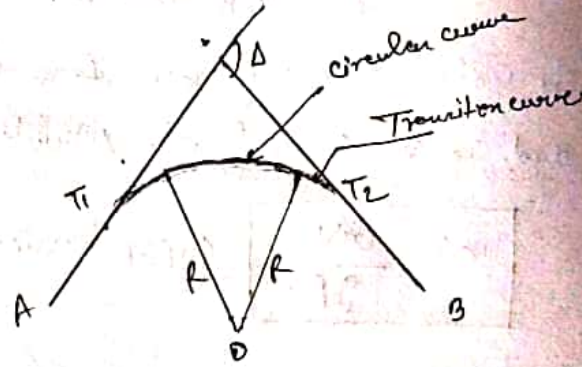
• for railway : $v = \sqrt{\frac{gR}{8}}$

TRANSITION CURVE

It is a non circular arc of varying curvature whose radius changes from a very great value at one end and some definite value at its junction with the curve at the other end. This curve is introduced b/w a straight and a circular or b/w two arcs of compound curve. These are provided for the purpose of giving easy changes of direction of a route. This curve is also called easement curves.

TYPES :-

1. A Cubic parabola.
2. A Spiral or clothoid.
3. A Lemniscate.



ADVANTAGES

- Eliminates the discomfort to passengers in the vehicles.
- Possibility of overturning and sliding of vehicles is eliminated.
- There will be less wear upon running gear.
- The transition from the tangent to circular curve and from circular curve to the tangent is made gradual.

Need/Conditions to be fulfilled by transition curve:-

- The transition curve should be tangential to the original straight and circular curve.
- The curvature of transition curve should be zero at the origin on straight line.
- The length of transition curve should be such that at the junction with the circular curve full super elevation is attained.
- The curvature along the transition curve should inc at the same rate as the superelevation.
- Its radius at the junction with the circular curve should be equal to the radius of circular curve.

Temp.

LENGTH OF TRANSITION CURVE

BY ARBITRARY GRADIENT / BY ADOPTING DEFINITE RATE OF SUPERELEVATION :-
In this method super-elevation is provided at a uniform rate say 1 in n.

For super-elevation of h value, the transition curve will be
 $L = nh$ metres.

The value of n may vary from 300-1200.
and, h is amount of super-elevation in m.

$$\boxed{h = \frac{GV^2}{9R}} \quad \text{SUPER-ELEVATION}$$

By Time Rate Method :-

Let the time rate of application of super-elevation be 'a' cm/sec.

Length of transition curve L metres.

h = amount of super-elevation.

v = speed of vehicle in m/sec.

Time taken provide full v.e, $t = \frac{h}{a}$.

Distance travelled = $S \times t$.

$$L = v \times \frac{h}{a} \Rightarrow$$

$$\boxed{L = \frac{vh}{a}}$$

Rate of change of Radial Acceleration :-

Let the $M \& L$ short : " $a = 0.3 \text{ m/sec}^2$.

It gives comfort condition for the passengers.

If, v = speed of vehicle.

R = Radius of curve.

Then radial acceleration = $\frac{v^2}{R}$.

Rate of change of radial acceleration,

$$a = \frac{\frac{v^2}{R}}{t} = \frac{v^2}{Rt}$$

$$\boxed{t = \frac{v^2}{aR}} \quad - (1)$$

If L = length of transition curve.

$$t = \frac{L}{V} \quad \text{--- (2)}$$

from ① and ②.

$$\frac{V^2}{aR} = \frac{L}{V}$$

$$L = \frac{V^3}{aR} \Rightarrow \boxed{L = \frac{V^3}{0.3R}}$$

PROCEDURE FOR SETTING OUT TRANSITION CURVE BY TANGENTIAL OFFSETS

First of all calculate the total tangent length, length of transition curve, length of simple circular curve, change of point T_1, T_2 .

1. Measure the length of total tangent length from point of intersection of two tangents as calculated. Mark point T_1, T_2 .
2. Divide the length of transition curve into number of chords $x_1, x_2, x_3, \dots$ depending upon length of transition curve.
3. Calculate the off sets by formula $y = \frac{x^3}{6RL}$.
4. With zero end of tape at T_1 measure distance x_1 along $T_1 B$.
5. Measure $\perp$ distance y_1 from end of tape distance x_1 . Thus locating first point on the transition curve.
6. Repeat the process for $x_2, x_3, \dots$.

Check:

At $x = \frac{L}{2}$ the transition curve bisects the shift and at $x = L$ the $\perp$ offset = 4×5 .



VERTICAL CURVE

When two different gradient meet at a point along the road surface they are connected in a vertical plane by a curve to avoid impact and for maintaining clear visibility. Such a curve is called vertical curve. The meeting point of two gradient is called apex. These curves are generally provided in the form of:

→ Circular curve.

→ Parabolic curve.

Parabolic curve is preferred and is invariably used due to simplicity in calculation and best riding qualities.

* LENGTH OF VERTICAL CURVE

$$\frac{\text{Algebraic difference of two grade}}{\text{Rate of change of grade}}$$

$$\frac{g_1 - g_2}{r}$$

where g_1 and g_2 = % age grade and r is rate of change of grade.



GRADIENT

- The grades or gradients is expressed can two ways:-
1. As 1 in 'n' when 'n' is horizontal distance corresponding to 1m vertical distance.
Gradient may be rise or fall.
 2. As % age. i.e., 2%, 2.5%.

* Rate of change of grade (R)

Characteristic of parabolic curve:- is that the gradient changes from point to point on the curve but the rate of change of grade is constant in parabola. ★

for I class railway r is 0.06%/20m at summits and 0.03%/20m at valleys. Twice these value for II class.

TYPE OF VERTICAL CURVE

There are two types of vertical curve.

① Summit curve:

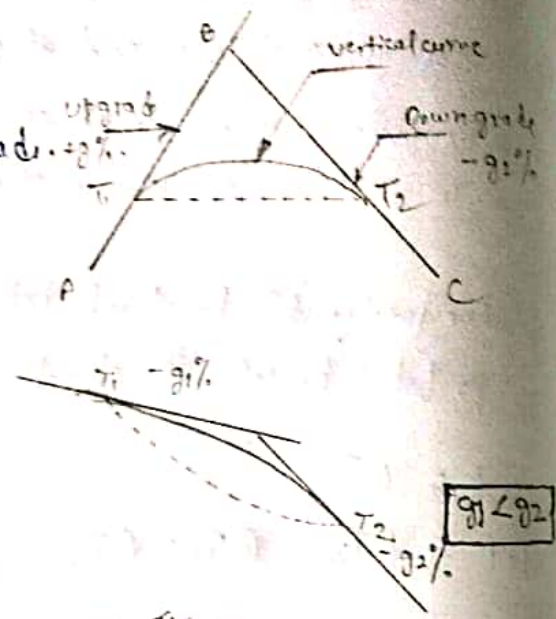
When the curve has a convexity upward, it is called summit or convex curve.

It is formed when:-

(i) Upgrade is followed by down grade.

(ii) when down grade $-g_1\%$ is followed by another down grade $-g_2\%$.

(iii) when upgrade $+g_1\%$ is followed by another upgrade $+g_2\%$.



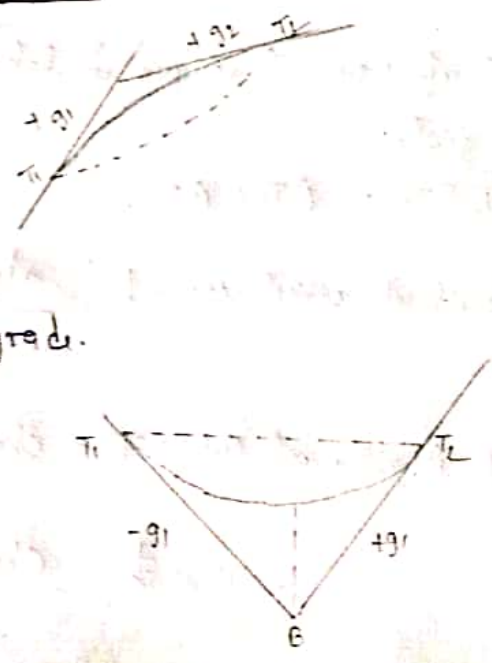
② Valley Curve:

The curve having concavity upward is called valley curve or concave curve.

(i) down grade is followed by upgrade.

(ii) $g_2 > g_1$

(iii) $g_2 < g_1$



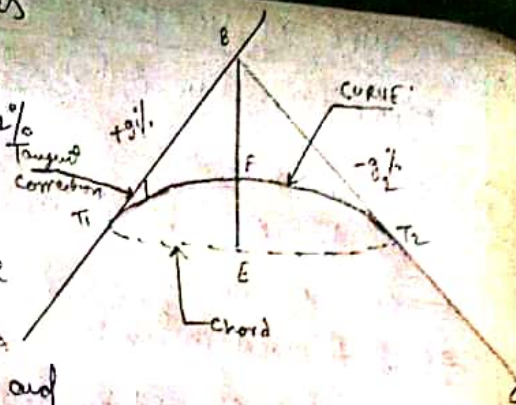
CHARACTERSTIC OF VERTICAL CURVE

AB and BC are two gradient lines meeting at point B.

$+g_1\%$ is gradient of line AB and $-g_2\%$ is gradient of line BC.

T_1 and T_2 is tangent points to the curve beginning and T_2 is end of curve.

E is the mid point of chord $T_1 T_2$ and F is " " " curve $T_1 F T_2$.



A line joining the point of intersection of the two tangents and vertex of parabola bisects the chord.

$$T_1 E = E T_2$$

$$T_1 F = F T_2 = B T_1 = B T_2.$$

The length of vertical curve is taken equal to sum of the length of two tangents.

$$\therefore T_1 F T_2 = B T_1 + B T_2.$$

$$\text{Length of vertical curve } L = \frac{\text{algebraic sum of two grade}}{\text{rate of change of grade.}}$$

Chainage of Beginning of curve T_1 = chainage of intersection point - half the length of V.C.

$$\Rightarrow \text{Chainage of } B = \frac{L}{2}$$

Chainage of ending of curve T_2 = chainage of B + $\frac{L}{2}$.

R.L of T_1 = R.L of B $\pm$ Level difference b/w B and T_1

$$\text{R.L of } T_1 = \text{R.L of } B \pm \frac{L}{2} \times \frac{g_1}{100}$$

Use + sign in g_1 is -ve and vice versa.

$$\text{R.L of } T_2 = \text{R.L of } B \pm \frac{L}{2} \times \frac{g_2}{100}$$

$$\text{R.L of } E = \frac{1}{2} (\text{R.L of } T_1 + \text{R.L of } T_2)$$