CURVES

Curves are regular bends provided in the lines of communication like roads, railways etc. and also in canals to bring about the gradual change in direction.

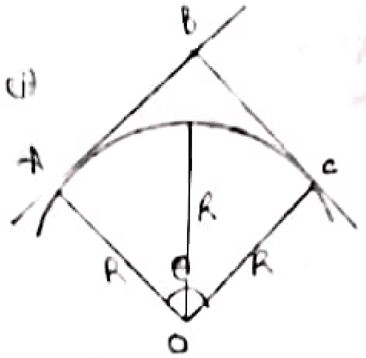
Horizontal curve — for gradual change in direction.

Vertical curve — for gradual change in gradient (ht).

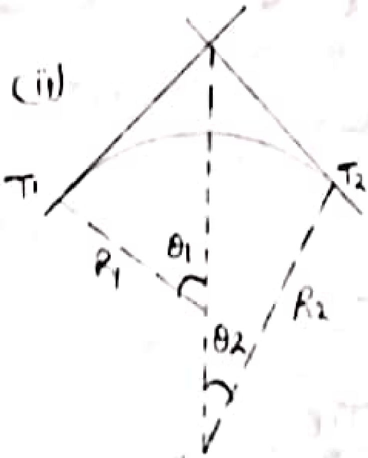
Types of curve —

(i) Horizontal curve — (i.) Simple circular curve (1 radius)
It is [illegible] when space [illegible] are not [illegible] forsing circle
(ii) Compound curve. (2 or more radius).
(iii.) Reverse curve (2 radii having curvature at diff. sides).
or serpentine

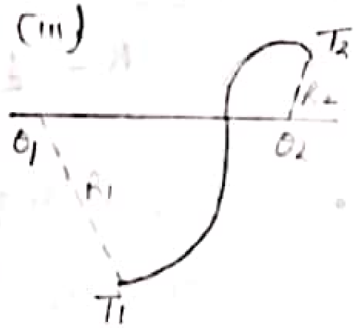
(i)



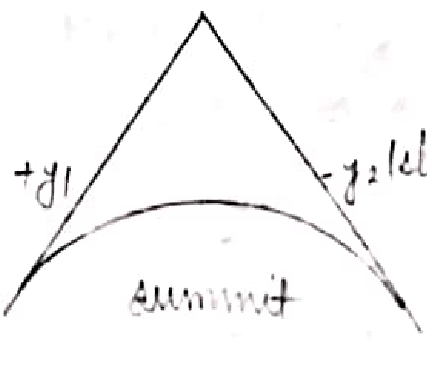
(ii)

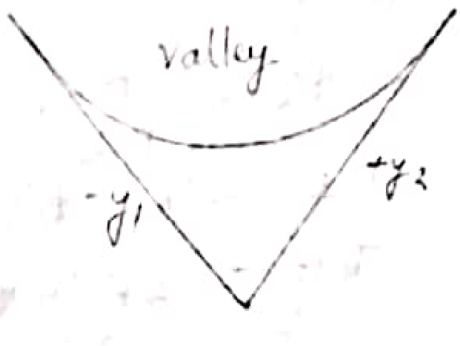


(iii)



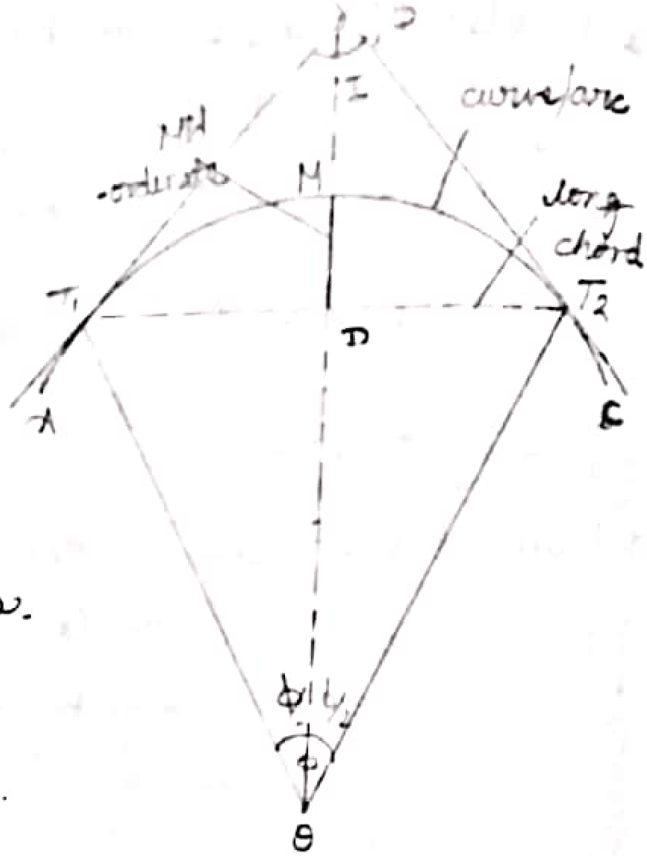
(ii) Vertical curve — (i.) Summit.
(ii.) Valley.





(iii) Transition curve – It is a curve of varying radius.

★ Elements of simple circular curve –



- T_1B = Ist tangent.
- BT_2 = IInd tangent.
- MD = mid-ordinate / versed sine of curve.
- BM = apex distance / external distance.
- ϕ = deflection angle.
- central angle, $T_1OT_2 = \phi$.
- radius of curvature, degree of curve

★ $$R = \frac{1719}{D}.$$

- tangent length (ΔOT_1B_1),

$$\tan \phi/2 = \frac{T_1B}{R}.$$

$$T_1B = R \tan \phi/2.$$

- length of long chord (T_1T_2), (T_1DO)

$$\sin \phi/2 = \frac{T_1D}{R}.$$

$$T_1D = R \sin \phi/2.$$

$$\therefore \quad T_1T_2 = 2R \sin \phi/2.$$

• curve length (T_1MT_2),

$$\theta = \frac{dx}{radius} = \frac{T_1MT_2}{R}.$$

$$\phi = \frac{T_1MT_2}{R}.$$

$$T_1MT_2 = \frac{R\phi}{\cancel{180°}}.$$

$$\therefore \quad T_1MT_2 = \frac{\pi R\phi}{180°}.$$

• Apex distance (BM),

In ΔOBT_1, $\cos\phi/2 = \frac{R}{OB}$.

$$OB = R\sec\phi/2.$$

$$\because \quad BM = OB - OM.$$

$$BM = R\sec\phi/2 - R.$$

$$BM = R(\sec\phi/2 - 1).$$

• Mid-ordinate (MD) –

In ODT_1, $\cos\phi/2 = \frac{OD}{OT_1}$.

$$\cos\phi/2 = \frac{OD}{R}.$$

$$OD = R\cos\phi/2.$$

$$OM = R.$$

$$MD = OM - OD.$$

$$= R - R\cos\phi/2.$$

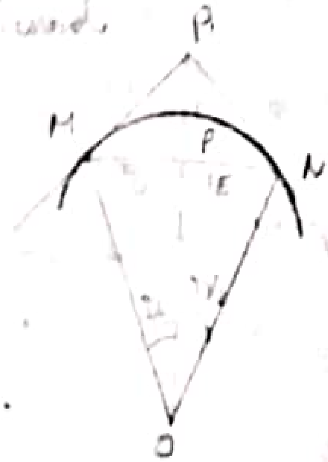
$$MD = R(1 - \cos\phi/2).$$

★ Relation between Radius of curvature (R) & degree of curve (D) –

MN = chord 30 m long, P = mid-pt of curve.

MP = MN = 15m.

$\sin D/2 = \frac{15}{R}$.



$R = \frac{15}{\sin D/2}$.

when D is small, $\sin D/2 = D/2$ radians.

$R = \frac{15}{\frac{D}{2} \times \frac{\pi}{180}} = \frac{360 \times 15}{D\pi}$.

$R = \frac{1718.87}{D}$ or $\boxed{R = \frac{1719}{D}}$.

★ Setting out of simple circular curve:–

1- Linear Method –

(i) Offset from the tangents.

→ radial offset –

BB1 = Ox = radial offset

$OB^2 = x^2 + R^2$.

$OB = \sqrt{x^2 + R^2}$.

$BB_1 = OB - OB_1$

$O_x = OB - \text{radius}$.

$\boxed{O_x = \sqrt{x^2 + R^2} - R}$.

→ Perpendicular offset –

ΔOPM, $R^2 = OP^2 + x^2$.

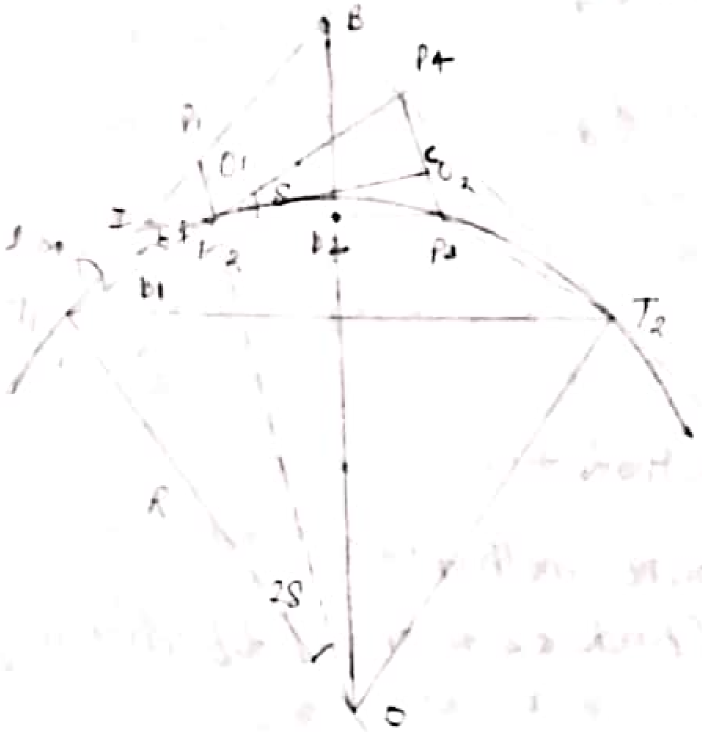
$OP = \sqrt{R^2 - x^2}$

$O_x = R - OP$.

$\boxed{O_x = R - \sqrt{R^2 - x^2}}$.

(ii) - Offset from long chord -

(iii) - Offset from chord produced -



$T_1P_1 = T_1P_2 = b_1$.

Arc, $T_1P_2 = R \times 2\delta$.

$b_1 = R \times 2\delta$.

$$\boxed{\delta = \frac{b_1}{2R}}.$$

$O_1 = b_1\delta$.

$$O_1 = \frac{b_1^2}{2R};$$

$O_2 = AC + BC$.

$BC = \frac{b_2^2}{2R}$. & $AC = \delta \times b_2$.

$= \frac{b_1 b_2}{2R}$.

$$O_2 = \frac{b_2^2}{2R} + \frac{b_1 b_2}{2R}.$$

$$O_2 = \frac{b_2}{2R}(b_1 + b_2).$$

(I). locate T_1 & T_2.
Find chainage.
1st & last sub-chord
(1) & offsets.

(II) Mark P_1 = 1st sub chord.

(III) at P_1, hang tape,
$O_1 = \frac{b_1^2}{2R}$.

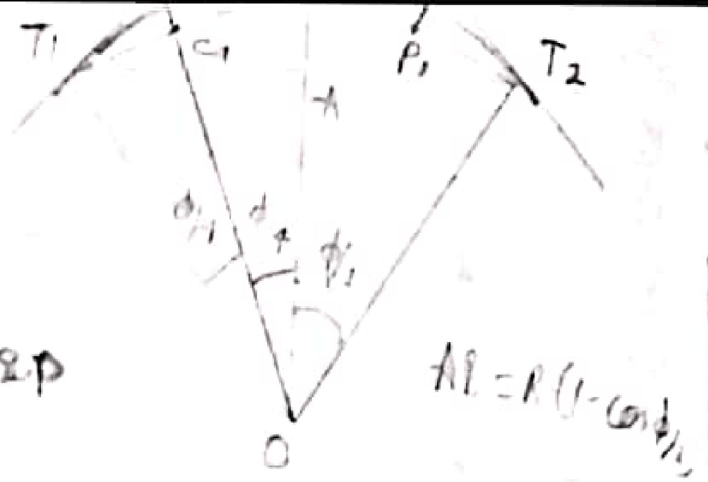
(IV.) Extend tape to P_4.

$R(1-\cos\phi/8)$.

T_1T_2. bisect it at A.

...ffect BA = $R(1-\cos\phi/_{1})$.

T_1B & T_2B, bisect them at C & D



...ular method –

Theodolite method: → by tangential angle

(Rankine's method / deflection angle method).

$= \frac{b_1}{2\delta_1}$. or $b_1 = \frac{\pi R \phi}{180}$. ($\phi = 2\delta$)

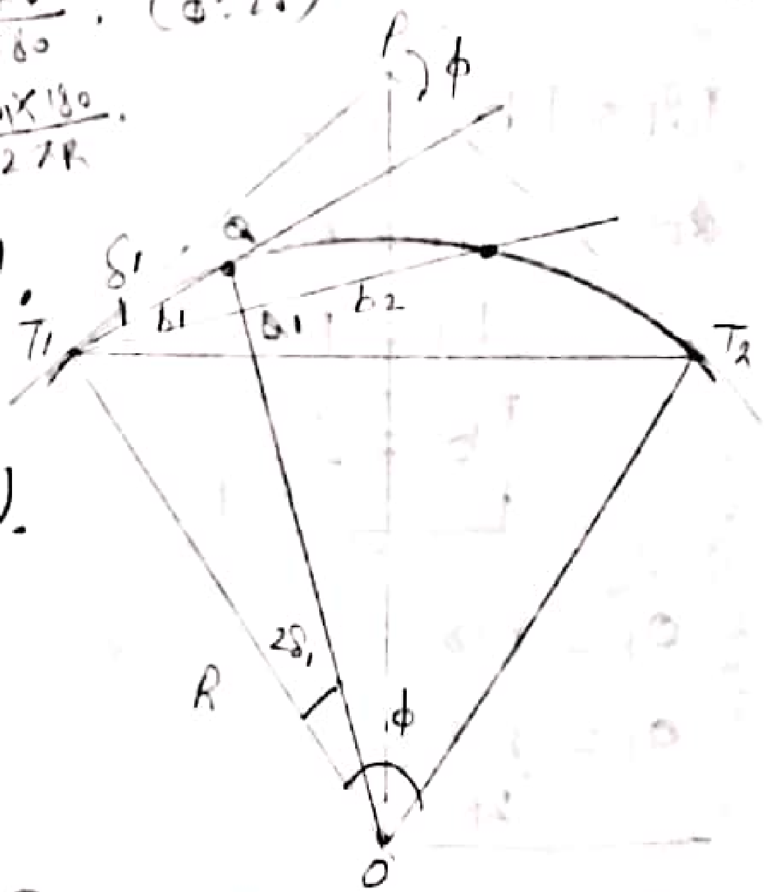
$\delta = \frac{b_1 \times 180}{2\pi R}$.

$= \frac{b_1 \times 360}{2\pi R \times 2}$ (degrees).

$= \frac{b_1 \times 360 \times 60}{4\pi R}$ (min.).

$= \frac{1718.87 b_1}{R}$ (min.).

$= \frac{1719 b_1}{R}$.



$\delta_2 = \delta_1 + \frac{1719 b_2}{R}$

...l deflection angle, $\delta = \delta_1 + \delta_2 + \delta_3 + \ldots\ldots \delta_n$.

...cate T_1 & T_2.

def

Transition curve – It is a curve of varying radius.

→ It is provided on both ends of a circular curve

→ radius of such a curve varies from infinity to a certain fixed value.

→ provided in railway tracks to ensure safe running of trains.

• Objects of Transition curve –

(i) for gradual change in direction.

(ii) to provide super elevation from 0 to a specific amount.

(iii) to avoid overturning of train. (vehicles).

(iv) to minimize wear & tear of vehicle.

(v) for comfort of passengers.

Types of transition curve –

ideal eq. for transition curve

1- Euler's spiral $\left(\phi = \frac{l^2}{2RL}\right)$.

2- Cubical spiral $\left(y = \frac{l^3}{6RL}\right)$.

l = dis. bet. two points
L = total length of transition curve.
R = radius of circular curve.

3- Lemniscate. curve $\left(r = \frac{P}{3\sin 2\alpha}\right)$ r = radius of curvature at any point.

4- Cubic Parabola $\left(y = \frac{x^3}{6RL}\right)$.

x = horizontal distance.
y = ⊥ distance.

length of transition curve
for roads

8/ Super-elevation — The height through which the outer edge of the road is raised is known as super-elevation / cant.

→ to balance the overturning force.

→ to balance centrifugal force.

when vehicle moves from straight to curved path, the centrifugal force tends to push the vehicle away from roads.

Th. eqn., $P = \frac{mv^2}{r}$

$\because m = \frac{W}{g}$ $\Rightarrow$ $P = \frac{Wv^2}{rg}$.

| P = centrifugal force
| v = speed (m/s)
| r = radius of curve (m)
| g = acceleration of gravity 9.81 m/s².

dividing by W →

$$\frac{P}{W} = \frac{v^2}{rg}.$$

where, $\frac{P}{W}$ = centrifugal ratio — ratio bet. centrifugal force & wt. of vehicle.

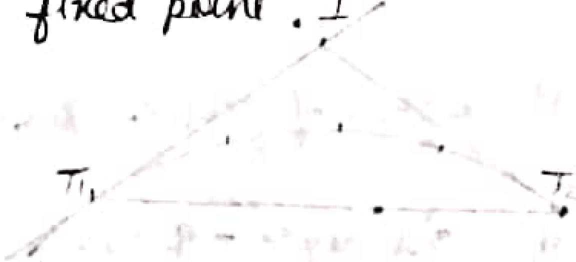
allowable limit of $\frac{P}{W}$, for roads $= 1/4$.

for railways $= 1/8$.

→ to prevent overturning of vehicle →

$W\sin\theta \geq P$.

• Chainage — distance from fixed point.



dist distance at which pegs fixed = 20.

no. of chord = ?

→ 1st chord & last chord length < 20m.

sum of chord = length of chord

① Radius of curve = 500 m, chord length = 30 m, chainage of intersection point = 950 m, angle of intersection = 120°. Find out:

(i) tangent length.

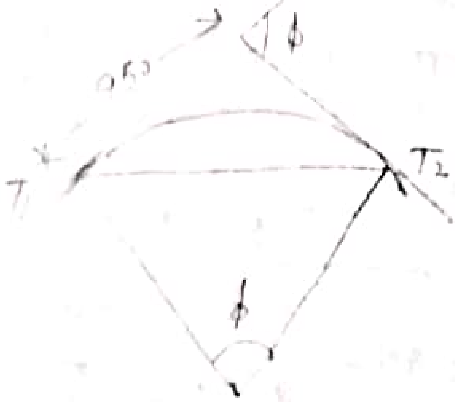
(ii) length of curve.

(iii) length of long chord.

(iv) degree of curve.

(v) no. of normal chord.

(vi) no. of sub-chord.



— deflection angle, $\phi = 180° - 120° = 60°$.

$R = 500m$.

(i) tangent length $= R\tan\phi/2 = 500 \cdot \tan(60°/2)$.

$= 288.68m$.

(ii) curve length $= \dfrac{\pi R\phi}{180°}$.

$= \dfrac{\pi \times 500 \times 60°}{180} = 523.598m$.

(iii) length of long chord $= 2R\sin\phi/2$.

$= 2 \times 500 \times \sin 30°$.

$= 500\ m$.

(iv) degree of curve, $D = \dfrac{1719}{R}$. $= 3.438°$.

(vi) chainage - 950m.

chainage of $T_1 = 950 - T_1P$.

$= 950 - 288.67$

$T_1 = 661.33$

chainage of T_2 = chainage of T_1 + length of curve.

$= 661.33 + 523.598.$

$= 1184.92.$

$\rightarrow T_1 = \frac{661.33}{30} = 22.04.$

22 full chain + $(30 \times 0.04) = 22 + 1.2$ m.

$\rightarrow T_2 = \frac{1184.92}{30} = 39.49.$

39 full chain + $(30 \times 0.49) = 39 + 14.7$ m.

(i) initial sub-chord $= 30 - 1.2.$ $= 28.8.$

I[st] peg $\rightarrow$ 23[rd] chain.

(ii) last sub-chord $= 14.7$ m.

(iv) no. of normal chord $= 39 - 23 = 16$ chord.

$\Sigma = (16 \times 30) + 28 + 14.7.$

$= 523.5$ m.

= length of chord.

normal chord = $39 - 23 = 16.$

16 normal + 2 sub-chords.

★ To calculate the length of transition curve –

(i) Adopting definite rate of super elevation :– 1 in 300 to 1 in 1200,

$L = nh$ metres.

where, L = length of transition curve, ~~x-section curve~~;

n = rate of super elevation in terms of 1 in n.

h = amount of super elevation.

① Calculate the rate of transition-curve required in order to obtain a super elevation of 10cm. assuming $n = 500$.

– $h = 10cm$, $n = 500$.

$l = nh = 500 \times 10 = 5000cm. = 50m.$

(ii) Time-rate method –

$L = \frac{vh}{a}$. where, L = length.

v = velocity m/s.

h = amount of super elevation.

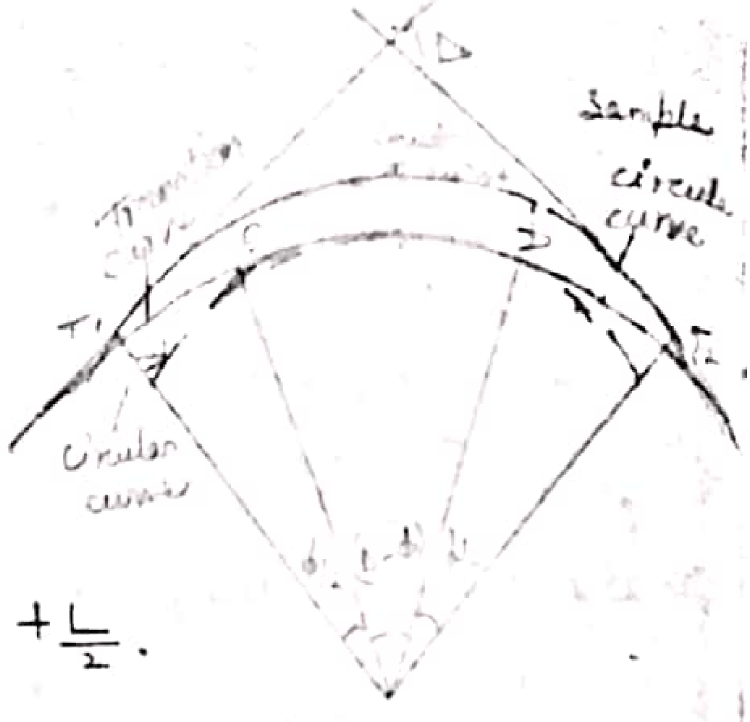
a = time-rate of super elevation.

= 2 cm/s.

= 2.5 - 5cm.

② Calculate the rate of transition curve to attain a max. super elevation of 12cm. Assuming a rate of super elevation of 3cm/s. The speed of vehicle is 6km/hr.

★ Combined curve —



shift of curve,

$$S = \frac{L^2}{24R}.$$

Tangent length,

$$T \cdot L = (R+S)\tan\frac{\Delta}{2} + \frac{L}{2}.$$

Length of curve,

$$L_{curve} = \frac{\pi R(\Delta - \phi)}{180} + 2L.$$

★★ Setting out of transition curve by tangential offset —

x - cubic curve —

$$y = \frac{x^3}{6RL}.$$

where, x - horizontal dis.

y = perpendicular distance.

R - Radius of circle curve.

→ Procedure —

(I.) (i) Calculate length of transition curve.

(ii) Total tangent length.

(iii) Total length of curve.

(II) Mark point T_1 & T_2.

(III) Divide the length of transition curve into no. of chords $x_1, x_2, x_3, \ldots$ using a peg interval.

(IV) Calculate the perpendicular offset by formula:

$$\frac{x^3}{6RL}.$$

(V) Plot the distance y, with the help of tape.

(Q) Two tangents meet at an angle 120°. A transition curve is introduced at both the ends of circular curve having 400m radius, chainage of intersection point is 1462m. Calculate the necessary data for setting out transition curve by the method of perpendicular offset. The length of transition curve is 90m. Peg interval = 20m.

— $L = 90m$, $R = 400m$,
Peg interval $= 20m$,

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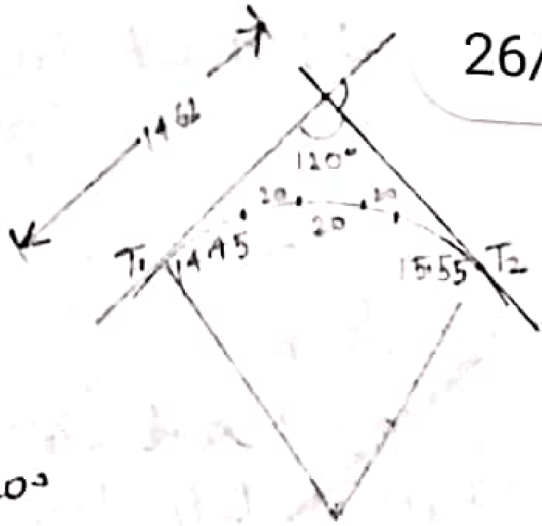

(i) $S = \dfrac{L^2}{24R}$.

$= \dfrac{90^2}{24 \times 400} = 0.844.$

Deflection angle, $\Delta = 180° - 120°$

$= 60°.$

(ii) Tangent length $=(R+S)\cdot\tan\frac{\Delta}{2}+\frac{L}{2}$

$$=(400+0.844)\cdot\tan\left(\frac{60}{2}\right)+\frac{L}{2}.$$

$$=276.43 \text{ m}.$$

(iii) chainage of T_1 = chainage of intersection pt. − tangent length

$$= 1462 - 276.43.$$

$$= 1185.57 \text{ m}.$$

no. of chains $= \frac{1185.57}{20} = 59.2785.$

59 full chains + (0.2785×20) m

59 full chains + 5.55 m.

∴ 1st Peg → at 60 chain.

initial sub chord, $x_1 = 20 - 5.55 = 14.45$m.

length of last sub-chord $= 90 - 74.45.$

$= 15.55$m.

∴ $x_1 = 14.55$m, $x_2 = 20$m, $x_3 = 20$m, $x_4 = 20$m,

$x_5 = 15.55$m.

no. of chords = 5

no. of full chords = 3.

" " sub chords = 2.

Perpendicular offset, $y = \frac{x^3}{6RL}$.

$x_1 = 14.15\,m.$

$$y_1 = \frac{(14.45)^3}{6\times400\times90} = 0.014\ m.$$

$$y_2 = \frac{(14.45+20)^3}{6\times400\times90} = 0.189\,m.$$

$$y_3 = \frac{(14.45+20+20)^3}{6\times400\times90} = 0.747\,m.$$

$$y_4 = \frac{(14.45+20+20+20)^3}{6\times400\times90} = 1.916\,m.$$

$$y_5 = \frac{90^3}{6\times400\times90} = 3.375\,m.$$

* **Vertical curves** – vertical curves are introduced at changes of gradient to avoid impact & to obtain good visibility.

These are set out in a vertical plane to round off the angle & to obtain gradual change of gradient.

1) **Valley curve** –

(i) A down grade followed by an up grade.

(i)

(ii)

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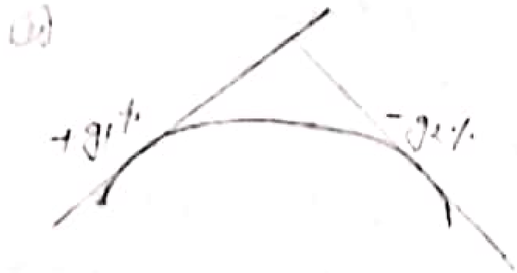
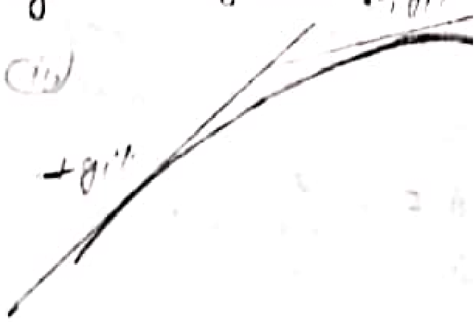
(ii) A down grade followed by down grade.

(iii) An up grade followed by up grade.

2)- <u>Summit curve</u> –

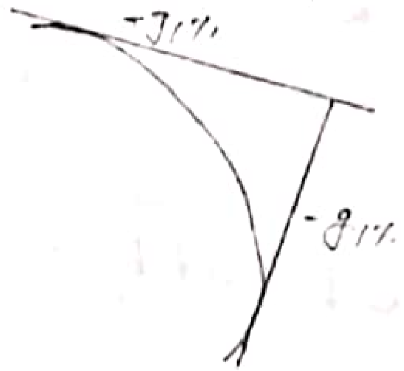
(i) An up grade followed by down grade.





(ii) An up grade followed by up grade.

(iii) A down grade followed by down grade.



★ Length of vertical curve –

$$L = \frac{\text{algebraic difference b/w gradient}}{\text{rate of change of gradient}}.$$

$$L = \frac{g_1 - g_2}{r}.$$

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Q A parabolic vertical curve is to be set out connecting two uniform grades of 0.8% & -0.6%, if the rate of change of grade is 0.1% per 30 m. Calculate the length of vertical curve.

$$\text{change of grade} = \frac{0.1\%}{30}.$$

$$g_1 = 0.8 - (-0.6)$$

$$= 1.4\%$$

$$L = \frac{1.4\%}{0.1\%/30} = 420m.$$

* Setting out of vertical curve.