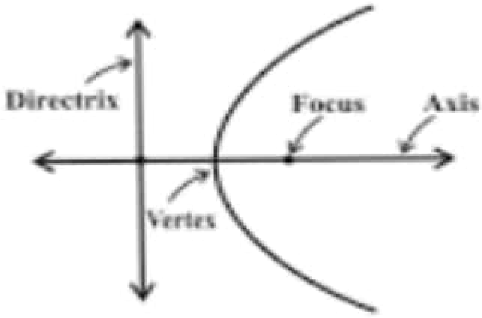
Parabola

A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.



(b)

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Fixed line is called **directrix** of parabola

Fixed point F is called the **focus**.

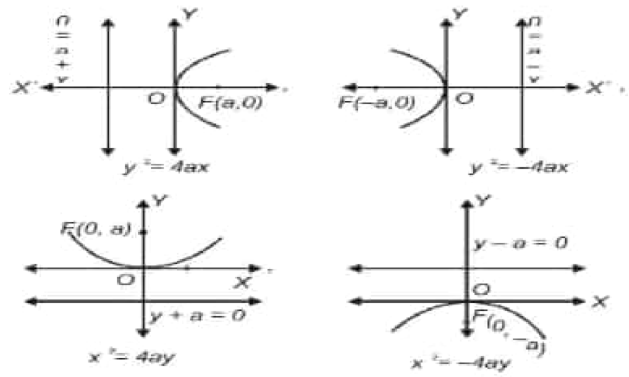
A line through focus & perpendicular to directrix is called **axis**.

Point of intersection of parabola with axis is called **vertex.**

Focal Chord Any chord passing through the focus.

Double Ordinate A chord perpendicular to the axis of a conic.

Latusrectum A double ordinate passing through the focus of the parabola.



Main facts about the Parabola

Equation	$y^2 = 4ax$ $(a > 0)$ Right hand	$y^2 = -4ax$ $a > 0$ Left hand	$x^2 = 4ay$ $a > 0$ Upwards	$x^2 = -4ay$ $a > 0$ Downwards
Axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Directrix	$x + a = 0$	$x - a = 0$	$y + a = 0$	$y - a = 0$
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Length of latus–rectum	$4a$	$4a$	$4a$	$4a$
Equation of latus–rectum	$x - a = 0$	$x + a = 0$	$y - a = 0$	$y + a = 0$

- **Latus Rectum:** A chord through focus perpendicular to axis of parabola is called its latus rectum.

Other Forms of a Parabola

If the vertex of the parabola is at a point A(h , k) and its latusrectum is of length 4a, then its equation is

1. $(y - k)^2 = 4a (x - h)$, its axis is parallel to OX i.e., parabola open rightward.
2. $(y - k)^2 = - 4a (x - h)$, its axis is parallel to OX' i.e., parabola open leftward.
3. $(x - h)^2 = - 4a (y - k)$, its axis is parallel to OY i.e., parabola open upward.
4. $(x - h)^2 = - 4a (y - k)$, its axis is parallel to OY' i.e., parabola open downward.

5. The general equation of a parabola whose axis is parallel to X – axis is $x = ay^2 + by + c$ and the general equation of a parabola whose axis is parallel to Y-axis is $y = ax^2 + bx + c$.

Equation of Tangent

A line which touches only one point of a parabola.

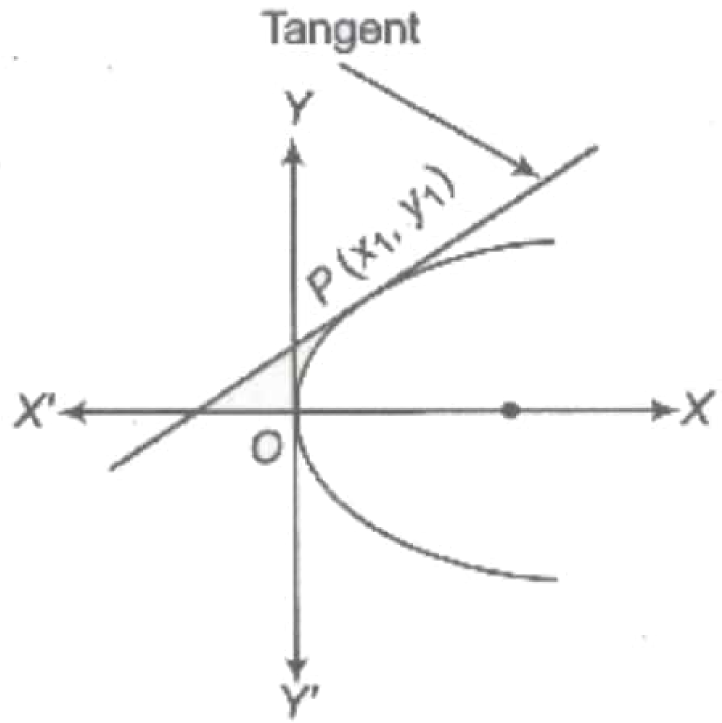
(i) **Point Form**

The equation of the tangent to the parabola $y^2 = 4ax$ at a point (x_1, y_1) is given by $yy_1 = 2a(x + x_1)$

(ii) **Slope Form**

(a) The equation of the tangent of slope m to the parabola $y^2 = 4ax$ is

$y = (mx + a/m)$



(b) The equation of the tangent of slope m to the parabola $(y - k)^2 = 4a (x - h)$ is given by

$(y - k)^2 = m (x — h) + a/m$

The coordinates of the point of contact are

$$\left(h + \frac{a}{m^2}, k + \frac{2a}{m}\right)$$

(iii) **Parametric Form**

The equation of the tangent to the parabola $y^2 = 4ax$ at a point $(at^2, 2at)$ is $yt = x + at^2$

(iv) The line $y = mx + c$ touches a parabola, if $c = a/m$ and the point of contact is

$$\left(\frac{a}{m^2}, \frac{2a}{m}\right).$$

Ellipse

Ellipse is the locus of a point in a plane which moves in such a way that the ratio of the distance from a fixed point (focus) in the same plane to its distance from a fixed straight line (directrix) is always constant, which is always less than unity.

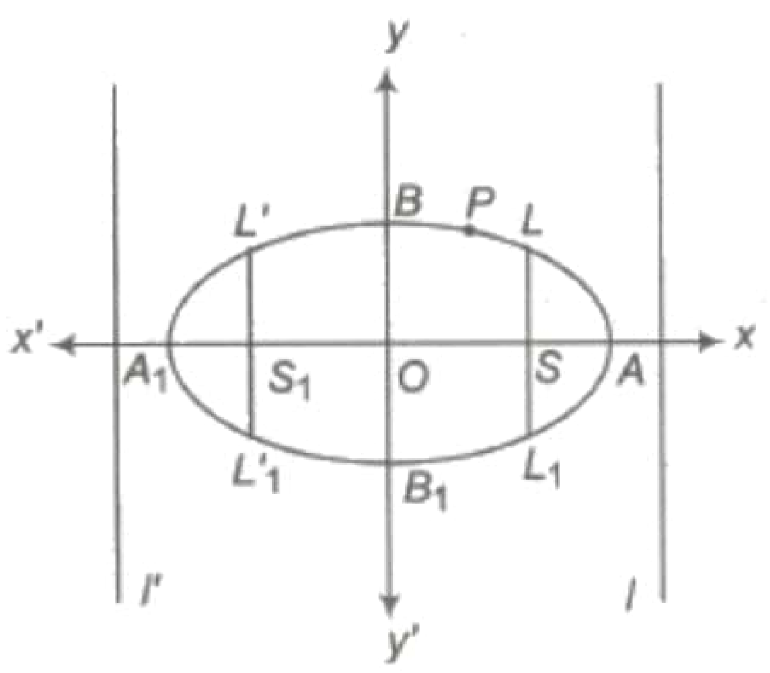
Major and Minor Axes

The line segment through the foci of the ellipse with its end points on the ellipse, is called its major axis.

The line segment through the centre and perpendicular to the major axis with its ended points on the ellipse, is called its minor axis.

Horizontal Ellipse i.e., $x^2 / a^2 + y^2 / b^2 = 1$, $0 < b < a$

If the coefficient of x^2 has the larger denominator, then its major axis lies along the x-axis, then it is said to be horizontal ellipse.



(i) Vertices $A(a, 0)$, $A_1(-a, 0)$

(ii) Centre $(0, 0)$

(iii) Major axis, $AA_1 = 2a$; Minor axis, $BB_1 = 2b$

(iv) Foci are $S(ae, 0)$ and $S_1(-ae, 0)$

(v) Directrices are $l : x = a / e$, l' ; $x = -a / e$

(vi) Latusrectum, $LL_1 = L' L_1' = 2b^2 / a$

(vii) Eccentricity, $e = \sqrt{1 - b^2 / a^2} < 1$

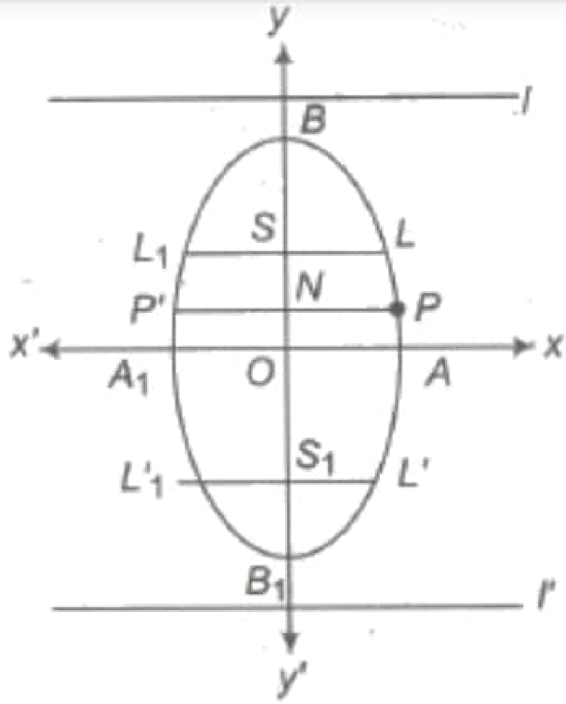
(viii) Focal distances are SP and S_1P i.e., $a - ex$ and $a + ex$. Also, $SP + S_1P = 2a$ = major axis.

(ix) Distance between foci = $2ae$

(x) Distance between directrices = $2a / e$

Vertical Ellipse i.e., $x^2 / a^2 + y^2 / b^2 = 1, 0 < a < b$

If the coefficient of x^2 has the smaller denominator, then its major axis lies along the y-axis, then it is said to be vertical ellipse.



(i) Vertices B(O, b), B_l(0,- b)

(ii) Centre O(0,0)

(iii) Major axis BB_l = 2b; Minor axis AA_l = 2a

(iv) Foci are S(0, ae) and S_l(0, – ae)

(v) Directrices are l : y = b / e ; l' : y = – b / e

(vi) Latusrectum LL_l = $L'L_l'$ = $2a^2$ / b

(vii) Eccentricity e = √1 – a^2 / b^2 < 1

(viii) Focal distances are SP and S_lP .i.e., b – ex and b + ex axis.

Also, SP + S_lP = 2b = major axis.

(ix) Distance between foci = 2be

(x) Distance between directrices = $2b / e$

Parametric Equation

The equation $x = a \cos \varphi$, $y = b \sin \varphi$, taken together are called the parametric equations of the ellipse $x^2 / a^2 + y^2 / b^2 = 1$, where φ is any parameter.

Hyperbola

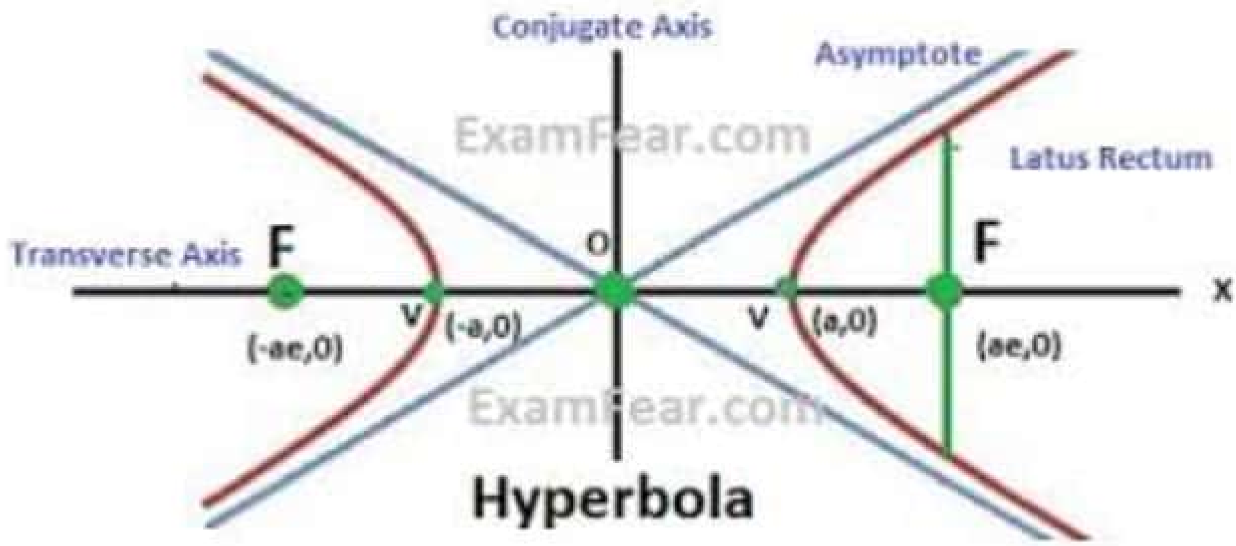
A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is constant. '*Difference*' means the distance to the 'farther' point minus the distance to the 'closer' point. The two fixed points are the foci and the mid-point of the line segment joining the foci is the center of the hyperbola.

The line through the foci is called the transverse axis. Also, the line through the center and perpendicular to the transverse axis is called the conjugate axis. The points at which the hyperbola intersects the transverse axis are called the vertices of the hyperbola.

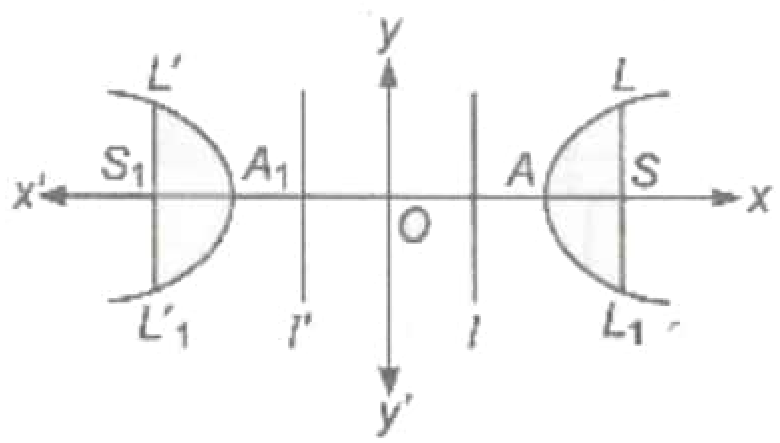
LATUS RECTUM OF A HYPERBOLA

The latus rectum of a hyperbola is a line segment perpendicular to the transverse axis,

through any of the foci with its end points lying on the hyperbola.



Hyperbola of the Form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



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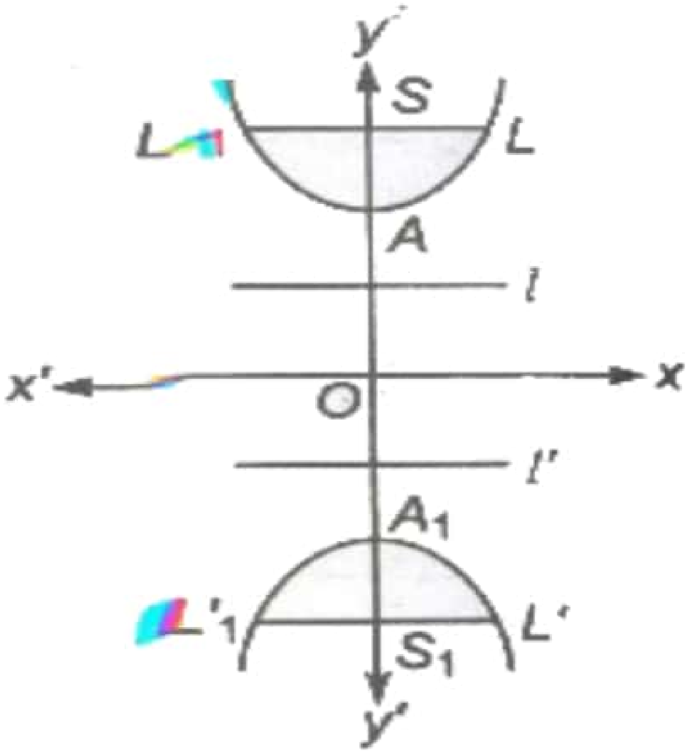
Conjugate hyperbola



Hyperbola / Imp. terms	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ or $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$
Centre	$(0,0)$	$(0,0)$
Length of transverse axis	$2a$	$2b$
Length of conjugate axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Equation of directrices	$x=\pm a/e$	$y=\pm b/e$
Eccentricity	$e=\sqrt{\left(\frac{a^2+b^2}{a^2}\right)}$	$e=\sqrt{\left(\frac{a^2+b^2}{b^2}\right)}$
Length of latus rectum	$2b^2/a$	$2a^2/b$
Parametric co-ordinates	$(a\sec\phi, b\tan\phi)$ $0\le\phi<2\pi$	$(b\sec\phi, a\tan\phi)$ $0\le\phi<2\pi$
Focal radii	$SP=ex_1-a$ $S'P=ex_1+a$	$SP=ey_1-b$ $S'P=ey_1+b$
Difference of focal radii $(S'P-SP)$	$2a$	$2b$
Tangents at the vertices	$x=-a, x=a$	$y=-b, y=b$
Equation of the transverse axis	$y=0$	$x=0$
Equation of the conjugate axis	$x=0$	$y=0$